Institute of Matematics

Winter semester:

Module I

[Analysis](#)	
[Differential Equations](#)	
[Geometry](#)	30 ECTS
[Linear Algebra](#)	
[Teaching mathematics with modern approaches](#)	

Summer semester:

Module I

[Algebra](#)	
[Differential Geometry](#)	
[Numerical Analysis](#)	30 ECTS
[Probability](#)	
[Topology](#)	

Course card

Course title	**Algebra**

Semester (winter/summer)	summer	ECTS	6

Lecturer(s)	Dr hab. Janusz Gwoździewicz, prof. UKEN Dr hab. Katarzyna Słomczyńska, prof. UKEN	

Department	Mathematics	

Course objectives (learning outcomes)

Introduction to the theory of groups, rings and fields.

Prerequisites

Knowledge	Complex numbers, foundations of the linear algebra (matrix theory).
Skills	Proficiency in numbers and matrix computations.
Courses completed	Introduction to Logic and Set Theory. Linear Algebra.

Course organization							
Form of classes	W (Lecture)	Group type					
		A (large group)	K (small group)	L (Lab)	S (Seminar)	P (Project)	E (Exam)
Contact hours			15				

Teaching methods:

Discussions and exercises.

Assessment methods:

	E – learning	Didactic games	Classes in schools	Field classes	Laboratory tasks	Individual project	Group project	Discussion participation	Student's presentation	Written assignment (essay)	Oral exam	Written exam	Other
										X			

Assessment criteria	Active participation in tutorials. Students have to write one or two essays.

Comments	

Course content (topic list)

1. Group theory: basic axioms and examples, groups of permutations, subgroups, cyclic groups, quotient groups and homomorphisms, Lagrange's Theorem, Isomorphism Theorems, finite abelian groups, group actions on sets, solvable groups.
2. Ring theory: basic definitions and examples, ideals and quotient rings, ring homomorphisms, Chinese Remainder Theorem, prime and maximal ideals, polynomial rings over fields.
3. Field theory: characteristic of a field, field of fractions, field extensions, algebraic and transcendental numbers, finite fields.

Compulsory reading

T. W. Hungerford, *Algebra*, Springer 1996

Recommended reading

http://www.jmilne.org/math/CourseNotes/GT.pdf
https://ww2.bc.edu/mark-reeder/Groups.pdf
http://www.jmilne.org/math/CourseNotes/FT.pdf
http://www1.spms.ntu.edu.sg/~frederique/chap2.pdf

Course card

Course title	**Analysis**

Semester (winter/summer)	winter	ECTS	6

Lecturer(s)	Dr hab. Paweł Wójcik, prof. UKEN	

Department	Mathematics	

Course objectives (learning outcomes)

Introduction to the theory and methods of analysis in one or two real variable. Introduction to the theory of measure.

Prerequisites

Knowledge	There are no prerequisites.
Skills	There are no prerequisites.
Courses completed	There are no prerequisites.

Course organization								
Form of classes	W (Lecture)	Group type						
		A (large group)	K (small group)	L (Lab)	S (Seminar)	P (Project)	E (Exam)	
Contact hours			15					

Teaching methods:

Discussions and exercises including computer usage (e.g. Maxima).

Assessment methods:

	E – learning	Didactic games	Classes in schools	Field classes	Laboratory tasks	Individual project	Group project	Discussion participation	Student's presentation	Written assignment (essay)	Oral exam	Written exam	Other
										X			

Assessment criteria	Students have to write one or two essays.

Comments	

Course content (topic list)

1. Real numbers and theirs subset.
2. Continuous functions.
3. Derivatives of functions with one variable.
4. Minimum, maximum and monotonicity of functions.
5. The Riemann integral.
6. Metric spaces (open, closed and compact subset).
7. Uniform convergence.
8. Derivatives of functions with two or three variables.
9. Minimum and maximum of functions with two variables
10. Lebesgue measure.and Lebesgue integral

Compulsory reading

Tao, T.: Analysis I, Hindustan Book Agency
https://lms.umb.sk/pluginfile.php/111477/mod_page/content/5/TerenceTao_Analysis.I.Third.Edition.pdf

Recommended reading
Rudin, W.: Principles of mathematical analysis, McGraw-Hill Science 1976

Course card

Course title	**Differential Equations**

Semester (winter/summer)	winter	ECTS	6

Lecturer(s)	Dr hab. Leszek Gasiński, prof. UKEN	

Department	Mathematics	

Course objectives (learning outcomes)

Introduction to the theory and methods of differential equations

Prerequisites

Knowledge	Differential calculus of functions of one and several variables. Integral calculus. Algebra of matrices and determinants.	
Skills	Calculation of derivatives of functions of one and several variables. Calculation of integrals.	
Courses completed	Mathematical Analysis. Linear Algebra.	

Course organization							
Form of classes	W (Lecture)	Group type					
		A (large group)	K (small group)	L (Lab)	S (Seminar)	P (Project)	E (Exam)
Contact hours			15				

Teaching methods:

Discussions and exercises.

Assessment methods:

	E – learning	Didactic games	Classes in schools	Field classes	Laboratory tasks	Individual project	Group project	Discussion participation	Student's presentation	Written assignment (essay)	Oral exam	Written exam	Other
										X			

Assessment criteria	Students have to write one or two essays.

Comments	

Course content (topic list)

1. Origin of Differential Equations
2. Differential Equations of First Order
3. Linear Differential Equations of Second Order
4. Linear Partial Differential Equations of First Order

Compulsory reading

1. W. Walter, *Ordinary Differential Equations*. *Graduate Texts in Mathematics*: Springer, 1998.
2. F. Ayres, JR.*, Theory and Problems of Differential Equations*: Schaum's Outline Series, McGraw-Hill Book Company, New York, St. Louis, San Francisco, Toronto, Sydney, 1952.

Recommended reading
1. B. Spain, *Ordinary Differential Equations*: Van Nostrand Reinhold Company, London, New York, Toronto, Melbourne, 1969.
2. Shepley L. Ross, *Differential Equations*: Blaisdell Publishing Company, New York, Toronto, London, 1964.

Course card

Course title	**Differential Geometry**

Semester (winter/summer)	summer	ECTS	6

Lecturer(s)	Dr hab. Justyna Szpond, prof. UKEN Prof. dr hab. Tomasz Szemberg	

Department	Mathematics	

Course objectives (learning outcomes)

Introduction to the theory of curves and surfaces, and methods of differential geometry.

Prerequisites

Knowledge	Elements of algebra and vector analysis. Vector spaces, linear and multilinear mappings. Calculus of functions of several variables.
Skills	Calculating of derivatives of functions of several variables.
Courses completed	Introduction to Logic and Set Theory Mathematical Analysis Linear Algebra Geometry

Course organization							
Form of classes	W (Lecture)	Group type					
		A (large group)	K (small group)	L (Lab)	S (Seminar)	P (Project)	E (Exam)
Contact hours			15			15	

Teaching methods:

Discussions and exercises.

Assessment methods:

	E – learning	Didactic games	Classes in schools	Field classes	Laboratory tasks	Individual project	Group project	Discussion participation	Student's presentation	Written assignment (essay)	Oral exam	Written exam	Other
										X			

Assessment criteria	Students have to write one or two essays.

Comments	

Course content (topic list)

1. Curves
1. Examples, Arclength Paramatrization
2. Local Theory: Frenet Frame
3. Some Global Results

2. Surfaces
1. Parametrized Surfaces and the First Fundamental Form
2. The Gauss Map and the Second Fundamental Form
3. The Codazzi and Gauss Equations and the Fundamental Theorem of Surface Theory

Compulsory reading

T. Shifrin, *Differential Geometry*: A First Course on Curves and Surfaces
available free at: *www.math.uga.edu/~shifrin/ShifrinDiffGeo.pdf*

Recommended reading
Michael David Spivak, *A Comprehensive Introduction to Differential Geometry*. Publish or Perish
2005

Course card

Course title	**Geometry**

Semester (winter/summer)	winter	ECTS	6

Lecturer(s)	Prof. dr hab. Tomasz Szemberg Dr hab. Justyna Szpond, prof. UKEN	

Department	Mathematics	

Course objectives (learning outcomes)

Introduction to the theory and methods of elementary geometry.

Prerequisites

Knowledge	There are no prerequisites.
Skills	Plotting points and figures in Cartesian coordinates.
Courses completed	There are no prerequisites.

Course organization								
Form of classes	W (Lecture)	Group type						
		A (large group)	K (small group)	L (Lab)	S (Seminar)		P (Project)	E (Exam)
Contact hours			15				15	

Teaching methods:

Discussions and exercises.

Assessment methods:

	E – learning	Didactic games	Classes in schools	Field classes	Laboratory tasks	Individual project	Group project	Discussion participation	Student's presentation	Written assignment (essay)	Oral exam	Written exam	Other
										X			

Assessment criteria	Students have to write an essay.

Comments	

Course content (topic list)

1. Triangles.
2. Isometries in the plane.
3. Similarities.
4. Circles and spheres.
5. Coordinates.
6. Complex numbers.

Compulsory reading

Coxeter, H.S.M.: Introduction to geometry, Wiley 1969

Recommended reading
Coxeter, H.S.M., Greizer, S.M.: Geometry revisited, The Mathematical Association of America 1967

Course card

Course title	**Linear Algebra**

Semester (winter/summer)	winter	ECTS	6

Lecturer(s)	Dr Karol Gryszka	

Department	Institute of Mathematics	

Course objectives (learning outcomes)

The aim of the course is to familiarize students with the main concepts and terminology of linear algebra. Emphasis is given to systems of linear equations, matrices, and vector spaces, determinants, eigenvalues, orthogonality and symmetric matrices.

Prerequisites

Knowledge	There are no prerequisites
Skills	There are no prerequisites
Courses completed	There are no prerequisites

Course organization							
Form of classes	W (Lecture)	Group type					
		A (large group)	K (small group)	L (Lab)	S (Seminar)	P (Project)	E (Exam)
Contact hours			15				

Teaching methods:

Reading course, tutorials.

Assessment methods:

	E – learning	Didactic games	Classes in schools	Field classes	Laboratory tasks	Individual project	Group project	Discussion participation	Student's presentation	Written assignment (essay)	Oral exam	Written exam	Other
								X		X			

Assessment criteria	Active participation in tutorials. Submitted written assignment.

Comments	

Course content (topic list)

1. Matrices, matrix operations and inverses.
2. Systems of linear equations.
3. Vector spaces - definition and examples.
4. Linear independence, basis and dimension.
5. Linear maps.
6. Range space and null space.
7. Representing linear maps with matrices.
8. Determinants - properties of determinants, the permutation expansion.
9. Eigenvalues and eigenvectors, invariant subspaces.
10. Diagonalizability.
11. Inner product, orthonormal bases.
12. Orthogonal projection into a line, geometric view of orthogonal projections.
13. Gram-Schmidt orthogonalization.
14. Symmetric matrices.

Compulsory reading

1. Sheldon Axler - Linear Algebra Done Right, Undergraduate Texts in Mathematics, Springer
2. Jim Hefferon - Linear Algebra (available free at: http://joshua.smcvt.edu/linearalgebra)

Recommended reading
David C. Lay - Linear Algebra and Its Applications, Pearson

Course card

Course title	**Numerical Analysis**

Semester (winter/summer)	summer	ECTS	6

Lecturer(s)	Dr Zbigniew Leśniak	

Department	Institute of Mathematics	

Course objectives (learning outcomes)

The aim of the course is to familiarize students with the basic concepts of analysis and implementations of algorithms for solving numerically the problems of continuous mathematics (as opposed to symbolic manipulations). Topics covered include: fundamental principles of digital computing and the implications for algorithm accuracy and stability. Emphasis is given to understanding the behaviour of numerical methods for solving linear algebra problems.

Prerequisites

Knowledge	Familiar with basics of linear algebra and real analysis.
Skills	Familiar with basics of Visual C#, C++ or Java.
Courses completed	There are no prerequisites.

Course organization												
Form of classes	W (Lecture)	Group type										
		A (large group)		K (small group)		L (Lab)		S (Seminar)		P (Project)		E (Exam)
Contact hours				15								

Teaching methods:

Reading course, tutorials.

Assessment methods:

	E – learning	Didactic games	Classes in schools	Field classes	Laboratory tasks	Individual project	Group project	Discussion participation	Student's presentation	Written assignment (essay)	Oral exam	Written exam	Other
								X		X			

Assessment criteria	Active participation in tutorials. Submitted written assignment.

Comments	

Course content (topic list)

15. Floating-point representation of numbers.
16. Finite precision arithmetic, the limits on the accuracy.
17. Round-off errors, truncation and discretization error.
18. Sensitivity of the solution of a problem to small changes in the data, ill-conditioned problems.
19. Backward error analysis, numerical stability of algorithms.
20. Interval arithmetic.
21. Numerically stable algorithms for computing values of functions.
22. Numerically stable algorithms for solving linear systems of equations.

Compulsory reading

1. Ward Cheney, David Kincaid - Numerical Mathematics and Computing, Thompson Brooks/Cole
2. Germund Dahlquist, Åke Björck - Numerical Methods in Scientific Computing, Volume 1, SIAM (working copy available for students enrolled in specific courses at: http://cristiancastrop.files.wordpress.com/2010/09/dahlquist-bjorck-vol-1.pdf)

Recommended reading

Åke Björck - Numerical Methods in Scientific Computing, *Volume 2*, SIAM

Course card

Course title	**Probability**

Semester (winter/summer)	summer	ECTS	6

Lecturer(s)	Dr Ireneusz Krech	

Department	Mathematics	

Course objectives (learning outcomes)

The course provides an introduction to the mathematical treatment of random phenomena occurring around us. The course will include the most fundamental as well as the most useful concepts and methods of the probability theory.

Prerequisites

Knowledge	There are no prerequisites.
Skills	There are no prerequisites.
Courses completed	There are no prerequisites.

Course organization								
Form of classes	W (Lecture)	Group type						
		A (large group)	K (small group)	L (Lab)	S (Seminar)	P (Project)	E (Exam)	
Contact hours			15					

Teaching methods:

Discussions and exercises.

Assessment methods:

	E – learning	Didactic games	Classes in schools	Field classes	Laboratory tasks	Individual project	Group project	Discussion participation	Student's presentation	Written assignment (essay)	Oral exam	Written exam	Other
										✖			

Assessment criteria	Students have to write an essay.

Comments	

Course content (topic list)

1) Combinatorics.
2) Axioms of probability, sample spaces and event.
3) Conditional probability, Bayes formula.
4) Independence as probability.
5) Discrete random variables and expected value.
6) Expectation of a function of a random variable, variance.
7) Random variables: Bernoulli, Binomial, Poisson and other discrete random variables.
8) Expected value of sums of random variables, cumulative distribution function.
9) Continuous random variables, expectation and variance.
10) Jointly distributed random variables, sums of independent random variables.
11) Conditional distribution.
12) Properties of expectations, expectations of sums of random variables, moments.
13) Covariance, variance of sums, correlations.
14) Conditional expectation, moment generating functions.
15) Weak Law of Large Number, The Central Limit Theorem, The Strong Law of Large Number.

Compulsory reading

Ross S., *Introduction to probability models*, Elsevier, 2007.

Recommended reading
Grinstead Ch. M., Snell J. L., *Introduction to probability* (file pdf available on the authors' website)

Course card

Course title	**Teaching mathematics with modern approaches**

Semester (winter/summer)	winter	ECTS	6

Lecturer(s)	Dr Miroslawa Sajka	

Department	Department of Mathematics	

Course objectives (learning outcomes)

The aim of the course is to familiarise participants with modern approaches to the teaching of mathematics, based on the analysis of selected theories and practical examples of methods tested in practical learning environments. Participants will also learn about the different ways of introducing selected concepts and the different ways of developing mathematical thinking, using different approaches, in different countries and at different levels of education. The emphasis will be on the practical side - students will experience and analyse different possible and modern learning environments.

Prerequisites

Knowledge	Basic knowledge of Mathematics at the level of secondary school
Skills	Basic skills of Mathematics at the level of secondary school
Courses completed	There are no prerequisites

Course organization							
Form of classes	W (Lecture)	Group type					
		A (large group)	K (small group)	L (Lab)	S (Seminar)	P (Project)	E (Exam)
Contact hours			15				

Teaching methods:

Lectures, discussions, lesson simulations, analysis and development of teachers' instructions and lesson plans, analysis of lessons video recordings, students' presentations (projects), students' and pupils' written work analysis, textbooks analysis.

Assessment methods:

	E – learning	Didactic games	Classes in schools	Field classes	Laboratory tasks	Individual project	Group project	Discussion participation	Student's presentation	Written assignment (essay)	Oral exam	Written exam	Other
		X			X	X		X		X			

Assessment criteria	Active participation in classes and discussions, completing written assignments, active participation in laboratory tasks and didactic games. Students have to write one essay.

Comments	

Course content (topic list)

- Neurodidactic aspects of the learning process
- Constructivist and non-constructivist approaches to teaching mathematics
- Active-based teaching of mathematics using the example of secondary school teaching
- Planning and designing lessons
- Design principles of modern learning environments: (a) situatedness, (b) digital technology, (c) embodiment, (d) inquiry-based education - their theoretical analysis and examples from practice
- Active methods in mathematics teaching, mathematical activation vs. pseudo-active methods in mathematics teaching, examples of practical application of active methods that can be used in secondary mathematics teaching and their critical analysis
- Developing algebraic and functional thinking in students at different educational levels, a variety of examples considering all 4 design principles simultaneously

Compulsory reading

Freudenthal, H. (2002). Revisiting mathematics education. China Lectures. Kluwer Academic Publishers Dordrecht/Boston/London.

Recommended reading

1. Liljedahl, P. (2016a). Building thinking classrooms: Conditions for problem solving, [in:] P. Felmer, E. Pehkonen, J. Kilpatrick (red.), Posing and Solving Mathematical Problems. Research in Mathematics Education, Cham: Springer, s. 361-386
2. Sajka, M., Nawalaniec, B. (2014). „Narratione de recherche" method in mathematical education at the secondary school level in Poland. [in:] Pytlak, M. (ed.) Communication in the mathematical classroom, Wydawnictwo Uniwersytetu Rzeszowskiego, Rzeszów, s. 166-179
3. Online resources (e.g. www.funthink.eu) and papers selected according to students' interests

Course card

Course title	**Topology**

Semester (winter/summer)	summer	ECTS	6

Lecturer(s)	Dr hab. Jacek Chmieliński, prof. UKEN	

Department	Institute of Mathematics	

Course objectives (learning outcomes)

The aim of the course is to familiarize students with metric and topological spaces and their basic properties such as completeness, compactness and connectedness, to the extent enabling usage these concepts in other courses (e.g. in functional analysis).

Prerequisites

Knowledge	Familiar with the elementary set theory. Familiar with basics of calculus (the set of reals as a metric space, properties of functions, limits, continuity, etc.)
Skills	Able to compute the limit of a sequence of real numbers, verify continuity of a mapping.
Courses completed	Mathematical Analysis 1 (Calculus 1)

Course organization												
Form of classes	W (Lecture)	Group type										
		A (large group)		K (small group)		L (Lab)		S (Seminar)		P (Project)		E (Exam)
Contact hours				15								

Teaching methods:

Reading course, tutorials.

Assessment methods:

	E – learning	Didactic games	Classes in schools	Field classes	Laboratory tasks	Individual project	Group project	Discussion participation	Student's presentation	Written assignment (essay)	Oral exam	Written exam	Other
										X			

Assessment criteria	Active participation in tutorials. Submitted essay.

Comments	

Course content (topic list)

> 23. Metric spaces
> 24. Topological spaces – basic properties of sets
> 25. Basis, axioms of countability
> 26. Continuity and homeomorphisms, topological invariants
> 27. Axioms of separability
> 28. Completeness, compactness and connectedness

Compulsory reading

1. K. Kuratowski, *Introduction To Set Theory and Topology,* Part II, Pergamon Press & PWN, 1962, 1972.
2. M.A. Armstrong, *Basic Topology*, Undergraduate Texts in Mathematics. Springer-Verlag, New York-Berlin, 1983.

Recommended reading - courses available in the Internet e.g.:
1. T.W. Koerner, *Metric and Topological Spaces,* https://www.dpmms.cam.ac.uk/~twk/Top.pdf
2. A. Hatcher, *Notes on Introductory Point-Set Topology,*
https://www.math.cornell.edu/~hatcher/Top/TopNotes.pdf
3. S.A. Morris, *Topology without tears* (English and various translations),
http://www.topologywithouttears.net/