



UNIwersytet  
PEDAGOGICZNY  
IM. KOMISJI EDUKACJI  
NARODOWEJ W KRAKOWIE



## Institute of Mathematics

**Winter semester:**

### Module I

<a href="#">Analysis</a>	20 ECTS
<a href="#">Geometry</a>	
<a href="#">Differential Equations</a>	
<a href="#">Linear Algebra</a>	

**Summer semester:**

### Module I

<a href="#">Algebra</a>	20 ECTS
<a href="#">Differential Geometry</a>	
<a href="#">Numerical Analysis</a>	
<a href="#">Topology</a>	



### Course card

Course title	<b>Linear Algebra</b>
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Semester (winter/summer)	winter	ECTS	5
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Lecturer(s)	Dr Karol Gryzka
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Department	Institute of Mathematics
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#### Course objectives (learning outcomes)

The aim of the course is to familiarize students with the main concepts and terminology of linear algebra. Emphasis is given to systems of linear equations, matrices, and vector spaces, determinants, eigenvalues, orthogonality and symmetric matrices.

#### Prerequisites

Knowledge	There are no prerequisites
Skills	There are no prerequisites
Courses completed	There are no prerequisites

#### Course organization

Form of classes	W (Lecture)	Group type						
		A (large group)	K (small group)	L (Lab)	S (Seminar)	P (Project)	E (Exam)	
Contact hours			15					

#### Teaching methods:

Reading course, tutorials.

#### Assessment methods:



Other	Written exam	Oral exam	Written assignment (essay)	Student's presentation	Discussion participation	Group project	Individual project	Laboratory tasks	Field classes	Classes in schools	Didactic games	E – learning
			X		X							

Assessment criteria	Active participation in tutorials. Submitted written assignment.
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Comments	
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#### Course content (topic list)

<ol style="list-style-type: none"> <li>1. Matrices, matrix operations and inverses.</li> <li>2. Systems of linear equations.</li> <li>3. Vector spaces - definition and examples.</li> <li>4. Linear independence, basis and dimension.</li> <li>5. Linear maps.</li> <li>6. Range space and null space.</li> <li>7. Representing linear maps with matrices.</li> <li>8. Determinants - properties of determinants, the permutation expansion.</li> <li>9. Eigenvalues and eigenvectors, invariant subspaces.</li> <li>10. Diagonalizability.</li> <li>11. Inner product, orthonormal bases.</li> <li>12. Orthogonal projection into a line, geometric view of orthogonal projections.</li> <li>13. Gram-Schmidt orthogonalization.</li> <li>14. Symmetric matrices.</li> </ol>
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#### Compulsory reading

<ol style="list-style-type: none"> <li>1. Sheldon Axler - Linear Algebra Done Right, Undergraduate Texts in Mathematics, Springer</li> <li>2. Jim Hefferon - Linear Algebra (available free at: <a href="http://joshua.smcvt.edu/linearalgebra">http://joshua.smcvt.edu/linearalgebra</a>)</li> </ol>
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#### Recommended reading

David C. Lay - Linear Algebra and Its Applications, Pearson



### Course card

Course title	<b>Algebra</b>
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Semester (winter/summer)	summer	ECTS	5
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Lecturer(s)	dr hab. Katarzyna Słomczyńska, prof. UP dr hab. Janusz Gwoździewicz, prof. UP
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Department	Mathematics
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#### Course objectives (learning outcomes)

Introduction to the theory of groups, rings and fields.

#### Prerequisites

Knowledge	Complex numbers, foundations of the linear algebra (matrix theory).
Skills	Proficiency in numbers and matrix computations.
Courses completed	Introduction to Logic and Set Theory. Linear Algebra.

#### Course organization

Form of classes	W (Lecture)	Group type						
		A (large group)	K (small group)	L (Lab)	S (Seminar)	P (Project)	E (Exam)	
Contact hours			15					

#### Teaching methods:

Discussions and exercises.

#### Assessment methods:



Other	
Written exam	
Oral exam	
Written assignment (essay)	X
Student's presentation	
Discussion participation	
Group project	
Individual project	
Laboratory tasks	
Field classes	
Classes in schools	
Didactic games	
E – learning	

Assessment criteria	Active participation in tutorials. Students have to write one or two essays.
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Comments	
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#### Course content (topic list)

1. Group theory: basic axioms and examples, groups of permutations, subgroups, cyclic groups, quotient groups and homomorphisms, Lagrange's Theorem, Isomorphism Theorems, finite abelian groups, group actions on sets, solvable groups.
2. Ring theory: basic definitions and examples, ideals and quotient rings, ring homomorphisms, Chinese Remainder Theorem, prime and maximal ideals, polynomial rings over fields.
3. Field theory: characteristic of a field, field of fractions, field extensions, algebraic and transcendental numbers, finite fields.

#### Compulsory reading

T. W. Hungerford, *Algebra*, Springer 1996

#### Recommended reading

- <http://www.jmilne.org/math/CourseNotes/GT.pdf>
- <https://www2.bc.edu/mark-reeder/Groups.pdf>
- <http://www.jmilne.org/math/CourseNotes/FT.pdf>
- <http://www1.spms.ntu.edu.sg/~frederique/chap2.pdf>



### Course card

Course title	<b>Geometry</b>		
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Semester (winter/summer)	winter	ECTS	5
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Lecturer(s)	Prof. dr hab. Tomasz Szemberg Dr hab. Justyna Szpond	
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Department	Mathematics	
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#### Course objectives (learning outcomes)

Introduction to the theory and methods of elementary geometry.

#### Prerequisites

Knowledge	There are no prerequisites.
Skills	Plotting points and figures in Cartesian coordinates.
Courses completed	There are no prerequisites.

#### Course organization

Form of classes	W (Lecture)	Group type						
		A (large group)	K (small group)	L (Lab)	S (Seminar)	P (Project)	E (Exam)	
Contact hours			15			15		

#### Teaching methods:

Discussions and exercises.

#### Assessment methods:



Other	
Written exam	
Oral exam	
Written assignment (essay)	X
Student's presentation	
Discussion participation	
Group project	
Individual project	
Laboratory tasks	
Field classes	
Classes in schools	
Didactic games	
E-learning	

Assessment criteria Students have to write an essay.

Comments

Course content (topic list)

Triangles.  
Isometries in the plane.  
Similarities.  
Circles and spheres.  
Coordinates.  
Complex numbers.

Compulsory reading

Coxeter, H.S.M.: Introduction to geometry, Wiley 1969

Recommended reading

Coxeter, H.S.M., Greizer, S.M.: Geometry revisited, The Mathematical Association of America 1967



Course card

Course title	<b>Analysis</b>		
Semester (winter/summer)	winter	ECTS	5
Lecturer(s)	Dr Paweł Wójcik		
Department	Mathematics		

Course objectives (learning outcomes)

Introduction to the theory and methods of analysis in one or two real variable.  
Introduction to the theory of measure.

Prerequisites

Knowledge	There are no prerequisites.
Skills	There are no prerequisites.
Courses completed	There are no prerequisites.

Course organization								
Form of classes	W (Lecture)	Group type						
		A (large group)	K (small group)	L (Lab)	S (Seminar)	P (Project)	E (Exam)	
Contact hours			15					

Teaching methods:

Discussions and exercises including computer usage (e.g. Maxima).

Assessment methods:





Other	
Written exam	
Oral exam	
Written assignment (essay)	X
Student's presentation	
Discussion participation	
Group project	
Individual project	
Laboratory tasks	
Field classes	
Classes in schools	
Didactic games	
E – learning	

Assessment criteria Students have to write one or two essays.

Comments

#### Course content (topic list)

Real numbers and their subset.  
 Continuous functions.  
 Derivatives of functions with one variable.  
 Minimum, maximum and monotonicity of functions.  
 The Riemann integral.  
 Metric spaces (open, closed and compact subset).  
 Uniform convergence.  
 Derivatives of functions with two or three variables.  
 Minimum and maximum of functions with two variables  
 Lebesgue measure and Lebesgue integral

#### Compulsory reading

Tao, T.: Analysis I, Hindustan Book Agency  
[https://lms.umb.sk/pluginfile.php/111477/mod\\_page/content/5/TerenceTao\\_Analysis.I.Third.Edition.pdf](https://lms.umb.sk/pluginfile.php/111477/mod_page/content/5/TerenceTao_Analysis.I.Third.Edition.pdf)

#### Recommended reading

Rudin, W.: Principles of mathematical analysis, McGraw-Hill Science 1976



Course card

Course title	<b>Differential Equations</b>		
Semester (winter/summer)	winter	ECTS	5
Lecturer(s)	Dr hab. Leszek Gasiński, prof. UP		
Department	Mathematics		

Course objectives (learning outcomes)

Introduction to the theory and methods of differential equations

Prerequisites

Knowledge	Differential calculus of functions of one and several variables. Integral calculus. Algebra of matrices and determinants.
Skills	Calculation of derivatives of functions of one and several variables. Calculation of integrals.
Courses completed	Mathematical Analysis. Linear Algebra.

Course organization								
Form of classes	W (Lecture)	Group type						
		A (large group)	K (small group)	L (Lab)	S (Seminar)	P (Project)	E (Exam)	
Contact hours			15					

Teaching methods:

Discussions and exercises.

Assessment methods:



Other	
Written exam	
Oral exam	
Written assignment (essay)	X
Student's presentation	
Discussion participation	
Group project	
Individual project	
Laboratory tasks	
Field classes	
Classes in schools	
Didactic games	
E-learning	

Assessment criteria Students have to write one or two essays.

Comments

#### Course content (topic list)

1. Origin of Differential Equations
2. Differential Equations of First Order
3. Linear Differential Equations of Second Order
4. Linear Partial Differential Equations of First Order

#### Compulsory reading

W. Walter, *Ordinary Differential Equations. Graduate Texts in Mathematics*: Springer, 1998.  
F. Ayres, JR., *Theory and Problems of Differential Equations*: Schaum's Outline Series, McGraw-Hill Book Company, New York, St. Louis, San Francisco, Toronto, Sydney, 1952.

#### Recommended reading

B. Spain, *Ordinary Differential Equations*: Van Nostrand Reinhold Company, London, New York, Toronto, Melbourne, 1969.

Shepley L. Ross, *Differential Equations*: Blaisdell Publishing Company, New York, Toronto, London, 1964.



### Course card

Course title	<b>Differential Geometry</b>
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Semester (winter/summer)	summer	ECTS	5
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Lecturer(s)	Dr hab. Justyna Szpond Prof. dr hab. Tomasz Szemberg
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Department	Mathematics
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### Course objectives (learning outcomes)

Introduction to the theory of curves and surfaces, and methods of differential geometry.
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### Prerequisites

Knowledge	Elements of algebra and vector analysis. Vector spaces, linear and multilinear mappings. Calculus of functions of several variables.
Skills	Calculating of derivatives of functions of several variables.
Courses completed	Introduction to Logic and Set Theory Mathematical Analysis Linear Algebra Geometry

Course organization								
Form of classes	W (Lecture)	Group type						
		A (large group)	K (small group)	L (Lab)	S (Seminar)	P (Project)	E (Exam)	
Contact hours			15				15	

### Teaching methods:

Discussions and exercises.
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Assessment methods:

Other	Written exam	Oral exam	Written assignment (essay)	Student's presentation	Discussion participation	Group project	Individual project	Laboratory tasks	Field classes	Classes in schools	Didactic games	E – learning
			X									

Assessment criteria	Students have to write one or two essays.
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Comments	
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Course content (topic list)

<p>Curves</p> <ol style="list-style-type: none"> <li>1. Examples, Arclength Paramatrization</li> <li>2. Local Theory: Frenet Frame</li> <li>3. Some Global Results</li> </ol> <p>Surfaces</p> <ol style="list-style-type: none"> <li>1. Parametrized Surfaces and the First Fundamental Form</li> <li>2. The Gauss Map and the Second Fundamental Form</li> <li>3. The Codazzi and Gauss Equations and the Fundamental Theorem of Surface Theory</li> </ol>
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Compulsory reading

<p>T. Shifrin, <i>Differential Geometry: A First Course on Curves and Surfaces</i> available free at: <a href="http://www.math.uga.edu/~shifrin/ShifrinDiffGeo.pdf">www.math.uga.edu/~shifrin/ShifrinDiffGeo.pdf</a></p>
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Recommended reading

Michael David Spivak, *A Comprehensive Introduction to Differential Geometry*. Publish or Perish 2005



### Course card

Course title	<b>Algebra</b>		
Semester (winter/summer)	summer	ECTS	5
Lecturer(s)	dr hab. Katarzyna Słomczyńska, prof. UP dr hab. Janusz Gwoździewicz, prof. UP		
Department	Mathematics		

### Course objectives (learning outcomes)

Introduction to the theory of groups, rings and fields.

### Prerequisites

Knowledge	Complex numbers, foundations of the linear algebra (matrix theory).
Skills	Proficiency in numbers and matrix computations.
Courses completed	Introduction to Logic and Set Theory. Linear Algebra.

Course organization								
Form of classes	W (Lecture)	Group type						
		A (large group)	K (small group)	L (Lab)	S (Seminar)	P (Project)	E (Exam)	
Contact hours			15					

### Teaching methods:

Discussions and exercises.

### Assessment methods:



Other	Written exam	Oral exam	Written assignment (essay)	Student's presentation	Discussion participation	Group project	Individual project	Laboratory tasks	Field classes	Classes in schools	Didactic games	E – learning
			X									

Assessment criteria	Active participation in tutorials. Students have to write one or two essays.
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Comments	
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#### Course content (topic list)

<p>1. Group theory: basic axioms and examples, groups of permutations, subgroups, cyclic groups, quotient groups and homomorphisms, Lagrange's Theorem, Isomorphism Theorems, finite abelian groups, group actions on sets, solvable groups.</p> <p>2. Ring theory: basic definitions and examples, ideals and quotient rings, ring homomorphisms, Chinese Remainder Theorem, prime and maximal ideals, polynomial rings over fields.</p> <p>3. Field theory: characteristic of a field, field of fractions, field extensions, algebraic and transcendental numbers, finite fields.</p>
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#### Compulsory reading

T. W. Hungerford, <i>Algebra</i> , Springer 1996
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#### Recommended reading

- <http://www.jmilne.org/math/CourseNotes/GT.pdf>
- <https://www2.bc.edu/mark-reeder/Groups.pdf>
- <http://www.jmilne.org/math/CourseNotes/FT.pdf>
- <http://www1.spms.ntu.edu.sg/~frederique/chap2.pdf>



### Course card

Course title	<b>Differential Geometry</b>		
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Semester (winter/summer)	summer	ECTS	5
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Lecturer(s)	Dr hab. Justyna Szpond Prof. dr hab. Tomasz Szemberg
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Department	Mathematics
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#### Course objectives (learning outcomes)

Introduction to the theory of curves and surfaces, and methods of differential geometry.
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#### Prerequisites

Knowledge	Elements of algebra and vector analysis. Vector spaces, linear and multilinear mappings. Calculus of functions of several variables.
Skills	Calculating of derivatives of functions of several variables.
Courses completed	Introduction to Logic and Set Theory Mathematical Analysis Linear Algebra Geometry

Course organization								
Form of classes	W (Lecture)	Group type						
		A (large group)	K (small group)	L (Lab)	S (Seminar)	P (Project)	E (Exam)	
Contact hours			15			15		

#### Teaching methods:





Discussions and exercises.

Assessment methods:

Other	Written exam	Oral exam	Written assignment (essay)	Student's presentation	Discussion participation	Group project	Individual project	Laboratory tasks	Field classes	Classes in schools	Didactic games	E – learning
			X									

Assessment criteria Students have to write one or two essays.

Comments

Course content (topic list)

1. Curves
  1. Examples, Arclength Paramatrization
  2. Local Theory: Frenet Frame
  3. Some Global Results
  
2. Surfaces
  1. Parametrized Surfaces and the First Fundamental Form
  2. The Gauss Map and the Second Fundamental Form
  3. The Codazzi and Gauss Equations and the Fundamental Theorem of Surface Theory

Compulsory reading

T. Shifrin, *Differential Geometry: A First Course on Curves and Surfaces*  
available free at: [www.math.uga.edu/~shifrin/ShifrinDiffGeo.pdf](http://www.math.uga.edu/~shifrin/ShifrinDiffGeo.pdf)

Recommended reading

Michael David Spivak, *A Comprehensive Introduction to Differential Geometry*. Publish or Perish 2005



## Course card

Course title	<b>Numerical Analysis</b>		
Semester (winter/summer)	summer	ECTS	5
Lecturer(s)	dr Zbigniew Leśniak		
Department	Institute of Mathematics		

### Course objectives (learning outcomes)

The aim of the course is to familiarize students with the basic concepts of analysis and implementations of algorithms for solving numerically the problems of continuous mathematics (as opposed to symbolic manipulations). Topics covered include: fundamental principles of digital computing and the implications for algorithm accuracy and stability. Emphasis is given to understanding the behaviour of numerical methods for solving linear algebra problems.

### Prerequisites

Knowledge	Familiar with basics of linear algebra and real analysis.
Skills	Familiar with basics of Visual C#, C++ or Java.
Courses completed	There are no prerequisites.

Course organization								
Form of classes	W (Lecture)	Group type						
		A (large group)	K (small group)	L (Lab)	S (Seminar)	P (Project)	E (Exam)	
Contact hours			15					

### Teaching methods:



Reading course, tutorials.

Assessment methods:

Other	Written exam	Oral exam	Written assignment (essay)	Student's presentation	Discussion participation	Group project	Individual project	Laboratory tasks	Field classes	Classes in schools	Didactic games	E – learning
			X		X							

Assessment criteria Active participation in tutorials.  
Submitted written assignment.

Comments

Course content (topic list)

15. Floating-point representation of numbers.
16. Finite precision arithmetic, the limits on the accuracy.
17. Round-off errors, truncation and discretization error.
18. Sensitivity of the solution of a problem to small changes in the data, ill-conditioned problems.
19. Backward error analysis, numerical stability of algorithms.
20. Interval arithmetic.
21. Numerically stable algorithms for computing values of functions.
22. Numerically stable algorithms for solving linear systems of equations.

Compulsory reading

1. Ward Cheney, David Kincaid - Numerical Mathematics and Computing, Thompson Brooks/Cole
2. Germund Dahlquist, Åke Björck - Numerical Methods in Scientific Computing, Volume 1, SIAM  
(working copy available for students enrolled in specific courses at:  
<http://cristiancastrop.files.wordpress.com/2010/09/dahlquist-bjorck-vol-1.pdf> )

Recommended reading

Åke Björck - Numerical Methods in Scientific Computing, *Volume 2*, SIAM

Course card



Course title	<b>Topology</b>
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Semester (winter/summer)	summer	ECTS	5
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Lecturer(s)	Dr hab. Jacek Chmieliński, prof. UP
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Department	Institute of Mathematics
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### Course objectives (learning outcomes)

The aim of the course is to familiarize students with metric and topological spaces and their basic properties such as completeness, compactness and connectedness, to the extent enabling usage these concepts in other courses (e.g. in functional analysis).

### Prerequisites

Knowledge	Familiar with the elementary set theory. Familiar with basics of calculus (the set of reals as a metric space, properties of functions, limits, continuity, etc.)
Skills	Able to compute the limit of a sequence of real numbers, verify continuity of a mapping.
Courses completed	Mathematical Analysis 1 (Calculus 1)

Course organization								
Form of classes	W (Lecture)	Group type						
		A (large group)	K (small group)	L (Lab)	S (Seminar)	P (Project)	E (Exam)	
Contact hours			15					

### Teaching methods:

Reading course, tutorials.



Assessment methods:

Other	Written exam	Oral exam	Written assignment (essay)	Student's presentation	Discussion participation	Group project	Individual project	Laboratory tasks	Field classes	Classes in schools	Didactic games	E-learning
			X									

Assessment criteria	Active participation in tutorials. Submitted essay.
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Comments	
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Course content (topic list)

<ol style="list-style-type: none"> <li>1. Metric spaces</li> <li>2. Topological spaces – basic properties of sets</li> <li>3. Basis, axioms of countability</li> <li>4. Continuity and homeomorphisms, topological invariants</li> <li>5. Axioms of separability</li> <li>6. Completeness, compactness and connectedness</li> </ol>
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Compulsory reading

<ol style="list-style-type: none"> <li>1. K. Kuratowski, <i>Introduction To Set Theory and Topology</i>, Part II, Pergamon Press &amp; PWN, 1962, 1972.</li> <li>2. M.A. Armstrong, <i>Basic Topology</i>, Undergraduate Texts in Mathematics. Springer-Verlag, New York-Berlin, 1983.</li> </ol>
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Recommended reading - courses available in the Internet e.g.:

1. T.W. Koerner, *Metric and Topological Spaces*,  
<https://www.dpmms.cam.ac.uk/~twk/Top.pdf>
2. A. Hatcher, *Notes on Introductory Point-Set Topology*,  
<https://www.math.cornell.edu/~hatcher/Top/TopNotes.pdf>
3. S.A. Morris, *Topology without tears* (English and various translations),  
<http://www.topologywithouttears.net/>